III. *Solutio generalis altera præcedentis Problematis, ope Combinationum & Serierum infinitarum,* per D. Abr. de Moivre. Reg. Soc Sodalem.

Deſignationes.

SI *B* & *C* colluſores duo ſimul certent, ad deſignandum *B* victorem eſſe, *C* victum, ſcribatur *B C*; atque viciſſim ad deſignandum *C* victorem eſſe, *B* victum; ſcribatur *C B*: & ſic de cæteris.

Ponatur 1° *B* vincere *A*, certamenque concludi tribus ludis

B A
B C
B D

} Sic patet *B* victorem neceſſario evadere.

Ponatur 2° *B* vincere *A*, certamenque concludi quatuor ludis

B A
C B
C D
C A

} Sic patet *C* victorem neceſſario evadere.

Ponatur 3° *B* vincere *A*, certamenque concludi quinque ludis

B A	*B A*
*C B**	*B C*
D C	*D B*
D A	*D A*
D B	*D C*

} Sic patet *D* victorem neceſſario evadere, idque duplici modo.

Ponatur 4° *B* prima vice vincere *A*, certamenque concludi ſex ludis.

Z   *B A*

BA	*BA*	*BA*
CB	*CB**	*BC*
*DC**	*CD**	*DB*
AD	*AC*	*AC*
AB	*AB*	*AD*
AC	*AD*	*AB*

Sic patet *A* victorem neceſſario evadere, idque triplici modo.

Ponatur 5° certamen concludi ſeptem ludis, ponaturque ſemper *B* prima vice vincere ipſum *A*.

BA	*BA*	*BA*	*BA*	*BA*
CB	*CB*	*CB**	*BC*	*BC*
DC	*DC**	*CD*	*DB*	*DB*
*AD**	*DA*	*AC*	*AD**	*DA*
BA	*BD*	*BA*	*CA*	*CD*
BC	*BC*	*BD*	*CB*	*CB*
BD	*BA*	*BC*	*CD*	*CA*

Sic patet *B* vel *C* neceſſario victores evadere, *B* triplici modo, *C* duplici.

Ponatur 6° certamen concludi octo ludis,

BA	*BA*	*BA*	*BA*	*BA*	*BA*	*BA*	*BA*
CB	*CB*	*CB*	*CB*	*CB**	*BC*	*BC*	*BC*
DC	*DC*	*DC**	*CD*	*CD*	*DB*	*DB*	*DB*
AD	*AD**	*DA*	*AC*	*AC*	*AD*	*AD**	*DA*
*BA**	*AB*	*BD*	*BA**	*AB*	*CA**	*AC*	*CD*
CB	*CA*	*CB*	*DB*	*DA*	*BC*	*BA*	*BC*
CD	*CD*	*CA*	*DC*	*DB*	*BD*	*BD*	*BA*
CA	*CB*	*CD*	*DA*	*DC*	*BA*	*BC*	*BD*

Sic patet *C* victorem evadere triplici, *D* duplici, *B* triplici modo, *&c.*

Nunc ordine ſcribantur literæ quibus victores deſignantur.

3, | 1 *B*
4, | 1 *C*
5, | 2 *D*
6, | 3 *A*
7, | 3 *B* + 2 *C*
8, | 3 *C* + 2 *D* + 3 *B*.
9, | 3 *D* + 2 *A* + 3 *C* + 3 *D* + 2 *A*
10, | 3 *A* + 2 *B* + 3 *D*   3 *A* + 2 *B* + 3 *A* + 2 *C* + 3 *D*
&c.

Per-

Perſpecta illarum formatione, patebit 1° literam *B* in ordine aliquo ſemper toties reperiri, quoties *A* in ordine ultimo & penultimo reperitur: 2° *C* in ordine aliquo toties reperiri quoties *B* in ordine ultimo & *D* in penultimo reperiuntur: 3° *D* in ordine aliquo toties reperiri quoties *C* in ultimo & *B* in penultimo: 4° *A* in ordine aliquo ſemper toties reperiri quoties *D* in ordine ultimo & *C* in penultimo reperiuntur.

Sed numerus variationum dato cuilibet ludorum numero competens, duplus eſt numeri variationum omnium dato ludorum numero unitate diminuto competentis: adeoque Probabilitas quam habet Colluſor *B* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *A* ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat idem *A*, ut vinceret dato ludorum numero minus duobus: & ſic de cæteris.

Probabilitas quam habet *C*, ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *B*, ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat *D*, ut vinceret dato ludorum numero minus duobus.

Probabilitas quam habet *D* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *C*, ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat *B*, ut vinceret dato ludorum numero minus duobus.

Probabilitas quam habet *A* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *D*, ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat *C* ut vinceret dato ludorum numero minus duobus.

Ex jam obſervatis facile eſt componere Tabulam Probabilitatum, quas *B*, *C*, *D*, *A* habent ut victores evadant dato ludorum numero, atque etiam illorum ſortium ſeu expectationum.

Tabula Probabilitatum, &c.

		B	C	D	A
′	3	$\frac{1}{4}\times\overline{4+3p}$	—	—	—
″	4	—	$\frac{1}{8}\times\overline{4+4p}$	—	—
‴	5	—	—	$\frac{2}{16}\times\overline{4+5p}$	—
⁗	6	—	—	—	$\frac{3}{32}\times\overline{4+6p}$
V	7	$\frac{3}{64}\times\overline{4+7p}$	$\frac{2}{64}\times\overline{4+7p}$	—	—
V′	8	$\frac{3}{128}\times\overline{4+8p}$	$\frac{3}{128}\times\overline{4+8p}$	$\frac{2}{128}\times\overline{4+8p}$	—
V″	9	—	$\frac{3}{256}\times\overline{4+9p}$	$\frac{6}{256}\times\overline{4+9p}$	$\frac{4}{256}\times\overline{4+9p}$
V‴	10	$\frac{4}{512}\times\overline{4+10p}$	$\frac{2}{512}\times\overline{4+10p}$	$\frac{6}{512}\times\overline{4+10p}$	$\frac{9}{512}\times\overline{4+10p}$
′X	11	$\frac{13}{1024}\times\overline{4+11p}$	$\frac{10}{1024}\times\overline{4+11p}$	$\frac{2}{1024}\times\overline{4+11p}$	$\frac{9}{1024}\times\overline{4+11p}$
X	12	$\frac{18}{2048}\times\overline{4+12p}$	$\frac{19}{2048}\times\overline{4+12p}$	$\frac{14}{2048}\times\overline{4+12p}$	$\frac{4}{2048}\times\overline{4+12p}$
		&c.	&c.	&c.	&c.

Jam vero Series istæ sunt convergentes, adeoque singulæ summari possunt per vulgarem Arithmeticam ; & obtinebuntur vel summæ accuratæ si possint, vel saltem approximatæ, si non liceat, terminos multos adhibere.

Invenire summas Probabilitatum ad infinitum usque pergentium, quas Collusores habent ut victores evadant.

Sint Probabilitates omnes ipsius B ad infinitum, nempe

$$B' + B'' + B''' + B'''' + B^{V} + B^{VI} \text{ \&c.} = y$$

Probabilitates ipsius C

$$C' + C'' + C''' + C'''' + C^{V} + C^{VI} \text{ \&c.} = z$$

Probabilitates ipsius D

$$D' + D'' + D''' + D'''' D^{V} + D^{VI} \text{ \&c.} = v$$

Probabilitates ipsius A

$$A' + A'' + A''' + A'''' A^{V} + A^{VI} \text{ \&c.} = x$$

Scribantur autem in Scala perpendiculariter descendente, ad hunc modum.

$$\begin{aligned}
B' &= B' \\
B'' &= B'' \\
B' &= \tfrac{1}{2}A'' + \tfrac{1}{4}A' \\
B'''' &= \tfrac{1}{2}A''' + \tfrac{1}{4}A'' \\
B^{V} &= \tfrac{1}{2}A'''' + \tfrac{1}{4}A''' \\
B^{VI} &= \tfrac{1}{2}A^{V} + \tfrac{1}{4}A''''
\end{aligned}$$

Proinde $y = \frac{1}{4} + \frac{3}{4}x$.

Ergo $y = \frac{1}{4} + \frac{1}{2}x + \frac{1}{4}x$.

Demonstratio.

Etenim prima columna perpendicularis $= y$, ex Hypothesi
Est vero $A' + A'' + A''' + A'''' + A^{V}$ &c. $= x$, ex hypothesi;
Ergo $\frac{1}{2}A' + \frac{1}{2}A'' + \frac{1}{2}A''' + \frac{1}{2}A'''' + \frac{1}{2}A^{V}$ &c. $= \frac{1}{2}x$.
Proinde $\frac{1}{2}A'' + \frac{1}{2}A''' + \frac{1}{2}A'''' + \frac{1}{2}A^{V}$ &c. $= \frac{1}{2}x - \frac{1}{2}A'$.
Et $B' + B'' + \frac{1}{2}A'' + \frac{1}{2}A''' + \frac{1}{2}A''''$ &c. $= \frac{1}{2}x - \frac{1}{2}A' + B' + B''$.

Sed

Sed $\frac{1}{2}A' = \bar{o}, B'' = o$ & $B' = \frac{1}{4}$, ut patet ex Tabula.

Ergo secunda columna perpendicularis $= \frac{1}{4} + \frac{1}{2}x$.

Sed tertia columna perpendicularis $= \frac{1}{4}x$.

erit igitur $y = \frac{1}{4} + \frac{3}{4}x$.

Simili modo scribantur

$$\begin{aligned} C' &= C' \\ C'' &= C'' \\ C''' &= \tfrac{1}{2}B'' + \tfrac{1}{4}D' \\ C'''' &= \tfrac{1}{2}B''' + \tfrac{1}{4}D'' \\ C^{V} &= \tfrac{1}{2}B'''' + \tfrac{1}{4}D''' \\ C^{VI} &= \tfrac{1}{2}B^{V} + \tfrac{1}{4}D'''' \end{aligned}$$

&c.

hoc est $z = \frac{1}{2}y + \frac{1}{4}v$.

Ergo $z = \frac{1}{8} + \frac{1}{2}y - \frac{1}{8} + \frac{1}{4}v$.

Scribantur etiam

$$\begin{aligned} D' &= D' \\ D'' &= D'' \\ D''' &= \tfrac{1}{2}C'' + \tfrac{1}{4}B' \\ D'''' &= \tfrac{1}{2}C''' + \tfrac{1}{4}B'' \\ D^{V} &= \tfrac{1}{2}C'''' + \tfrac{1}{4}B''' \\ D^{VI} &= \tfrac{1}{2}C^{V} + \tfrac{1}{4}B'''' \end{aligned}$$

&c.

& pari Argumento patebit $v + \frac{1}{2}z + \frac{1}{4}y$

Scribantur denique

$$\begin{aligned} A' &= A' \\ A'' &= A'' \\ A''' &= \tfrac{1}{2}D'' + \tfrac{1}{4}C' \\ A'''' &= \tfrac{1}{2}D''' + \tfrac{1}{4}C'' \\ A^{V} &= \tfrac{1}{2}D'''' + \tfrac{1}{4}C''' \\ A^{VI} &= \tfrac{1}{2}D^{V} + \tfrac{1}{4}C'''' \end{aligned}$$

&c.

Unde concludetur $x = \frac{1}{2}v + \frac{1}{4}z$

Resolutis autem quatuor istis æquationibus, reperietur

$$B' + B'' + B''' + B'''' \&c. = y = \frac{56}{149}$$

$$C' + C'' + C''' + C'''' \&c. = z = \frac{36}{149}$$

$$D' + D'' + D''' + D'''' \&c. = v = \frac{32}{149}$$

$$A' + A'' + A''' + A'''' \&c. = x = \frac{25}{149}$$

Valoribus istis inventis, ponatur jam $\frac{56}{149} = b$, $\frac{36}{149} = c$, $\frac{32}{149} = d$, $\frac{25}{149} = a$.

Iterum sit.

$3 B' p + 4 B'' p + 5 B''' p + 6 B'''' p$ &c. $= p y$.
$3 C' p + 4 C'' p + 5 C''' p + 6 C'''' p$ &c. $= p z$.
$3 D' p + 4 D'' p + 5 D''' p + 6 D'''' p$ &c. $= p v$.
$3 A' p + 4 A'' p + 5 A''' p + 6 A'''' p$ &c. $= p x$.

$3 B' = 3 B'$
$4 B'' = 4 B''$
$5 B''' = \frac{5}{2} A'' + \frac{5}{4} A'$
$6 B'''' = \frac{6}{2} A''' + \frac{6}{4} A''$
$7 B^{V} = \frac{7}{2} A'''' + \frac{7}{4} A'''$
$8 B^{VI} = \frac{8}{2} A^{V} + \frac{8}{4} A''''$

Ergo $y = \frac{3}{4} + \frac{3}{4} x + a$

Etenim prima Columna perpendicularis $= y$, ex Hypothesi:
$3 B' + 4 B'' = \frac{3}{4}$: Nam est $B' = \frac{1}{4}$, & $B'' = 0$.
$3 A' + 4 A'' + 5 A'''$ &c. $= x$ ex Hypothesi.
$A' + A'' + A'''$ &c. $= a$, ut repertum est.

Est igitur $4 A' + 5 A'' + 6 A''' + 7 A''''$ &c. $= x + a$
Et $\frac{4}{2} A + \frac{5}{2} A'' + \frac{6}{2} A''' + \frac{7}{2} A''''$ &c. $= \frac{1}{2} x + \frac{1}{2} a$.

Sed

Sed $A' = 0$

Ergo ſecunda Columna perpendicularis $= \frac{3}{4} + \frac{1}{2} x + \frac{1}{2} a.$

$$3 A' + 4 A'' + 5 A''' + 6 A'''' \text{ \&c.} = x$$
$$2 A' + 2 A'' + 2 A''' + 2 A'''' \text{ \&c.} = 2 a$$

Eſt igitur $5 A' + 6 A'' + 7 A''' + 8 A''''$ &c. $= x + 2 a.$

Et $\frac{5}{4} A' + \frac{6}{4} A'' + \frac{7}{4} A''' + \frac{8}{4} A''''$ &c. $= \frac{1}{4} x + \frac{1}{2} a.$

Eſt igitur tertia Columna perpendicularis $= \frac{1}{4} x + \frac{1}{2} a.$

Erit igitur $y = \frac{3}{4} + \frac{1}{2} x + \frac{1}{2} a + \frac{1}{4} x + \frac{1}{2} a$

ſive $y = \frac{3}{4} + \frac{3}{4} x + a,$ quod erat probandum.

$$3 C' = 3 C'$$
$$4 C'' = 4 C''$$
$$5 C''' = \frac{5}{2} B'' + \frac{5}{4} D'$$
$$6 C'''' = \frac{6}{2} B''' + \frac{6}{4} D''$$
$$7 C^{V} = \frac{7}{2} B'''' + \frac{7}{4} D'''$$
$$8 C^{V'} = \frac{8}{2} B^{V} + \frac{8}{4} D''''$$

&c.

Ergo $z = \frac{1}{2} y + \frac{1}{2} b + \frac{1}{4} v + \frac{1}{2} d.$

Etenim prima Columna perpendicularis $= z$, ex Hypotheſi.

$$3 C' + 4 C'' = \tfrac{1}{2}.$$
$$3 B' + 4 B'' + 5 B''' + 6 B'''' \text{ \&c.} = y.$$
$$B' + B'' + B''' + B'''' \text{ \&c.} = b.$$

Eſt igitur $4 B' + 5 B'' + 6 B''' + 7 B''''$ &c. $= y + b.$

Sed $4 B' = 1.$

Ergo $5 B'' + 6 B''' + 7 B''''$ &c. $= y + b - 1.$

$\frac{5}{2} B'' + \frac{6}{2} B''' + \frac{7}{2} B''''$ &c. $= \frac{1}{2} y + \frac{1}{2} b - \frac{1}{2}.$

Ergo ſecunda Columna perpendicularis $= \frac{1}{2} + \frac{1}{2} y + \frac{1}{2} b - \frac{1}{2}$
$= \frac{1}{2} y + \frac{1}{2} b.$

Iterum, $3 D' + 4 D'' + 5 D''' + 6 D''''$ &c. $= v$

$2 D' + 2 D'' + 2 D''' + 2 D''''$ &c. $= 2d$

Eſt igitur $5 D' + 6 D'' + 7 D''' + 8 D''''$ &c. $= v + 2 d.$

Et $\frac{5}{4} D' + \frac{6}{4} D'' + \frac{7}{4} D''' + \frac{8}{4} D''''$ &c. $= \frac{1}{4} v + \frac{1}{2} d.$

Ergo tertia Columna perpendicularis $= \frac{1}{4} v + \frac{1}{2} d$

Eſt igitur $z = \frac{1}{2} y + \frac{1}{2} b + \frac{1}{4} v + \frac{1}{2} d,$ quod erat probandum.

Eodem prorſus ordine ſcribantur.

$3\ D' = 3\ D'$
$4\ D'' = 4\ D''$
$5\ D''' = \frac{5}{2}\ C'' + \frac{5}{4}\ B'$
$6\ D'''' = \frac{6}{2}\ C''' + \frac{6}{4}\ B''$
$7\ D^{V} = \frac{7}{2}\ C'''' + \frac{7}{4}\ B'''$
$8\ D^{VI} = \frac{8}{2}\ C^{V} + \frac{8}{4}\ B''''$
&c.

Unde $v = \frac{1}{2}z + \frac{1}{2}c + \frac{1}{4}y + \frac{1}{2}b$.

$3\ A' = 3\ A'$
$4\ A'' = 4\ A''$
$5\ A''' = \frac{5}{2}\ D'' + \frac{5}{4}\ C'$
$6\ A'''' = \frac{6}{2}\ D''' + \frac{6}{4}\ C''$
$7\ A^{V} = \frac{7}{2}\ D'''' + \frac{7}{4}\ C'''$
$8\ A^{VI} = \frac{8}{2}\ D^{V} + \frac{8}{4}\ C''''$
&c.

Et $x = \frac{1}{2}v + \frac{1}{2}d + \frac{1}{4}z + \frac{1}{2}c$.

Quæ quidem Concluſiones eodem modo demonſtrantur ac ſuperiores.

Solutis autem quatuor iſtis æquationibus, elicietur $y = \frac{45536}{149|^2}$, $z = \frac{38724}{149|^2}$, $v = \frac{37600}{149|^2}$ $x = \frac{33547}{149|^2} = \frac{33547}{22201}$

Ergo, ſi velint *B*, *C*, *D*, *A* vendere Spectatori cuidam *R* ſummas quas ſinguli obtinere ſperant, æquum erit ut emptor *R* pendat

ipſi *B* $4 \times \frac{56}{149} + \frac{45536}{22201}p$, ipſi *C* $4 \times \frac{36}{149} + \frac{38724}{22201}p$.

ipſi *D* $4 \times \frac{32}{149} + \frac{37600}{22201}p$, ipſi *A* $4 \times \frac{25}{149} + \frac{33547}{22201}p$.

Invenire Probabilitates quas habent B, C, D, A, ut mulctentur, dato ludorum numero.

Si Ludi duo tantum ſint, erunt hoc modo.

$\frac{B\ A}{C\ B}$	$\frac{B\ A}{B\ C}$	Unde patet *B* vel *C* neceſſario mulctari.

Si Ludi tres fuerint, hoc modo ſe res habet.

$\frac{B\ A}{C\ B}$	$\frac{B\ A}{C\ B}$	$\frac{B\ A}{B\ C}$	$\frac{B\ A}{B\ C}$	Hinc patet *C*, vel *D* vel *B* neceſſario mulctari.
D C	*C D*	*D B*	*B D*	

Si vero quatuor Ludi fuerint:

B A	*B A*	*B A*	*B A*	*B A*	*B A*	Debet igitur *A* triplici modo, *D* duplici, *C* simplici, mulctari.
C B	*C B*	*C B*	*C B*	*B C*	*B C*	
D C	*D C*	*C D*	*C D*	*D B*	*D B*	
A D	*D A*	*A C*	*C A*	*A D*	*D A*	

Et sic de cæteris. Ex quibus manifesta est Compositio Tabulæ subjunctæ Probabilitatum quas *B*, *C*, *D*, *A* habent ut mulctentur, dato ludorum numero.

	Num Lud.	B	C	D	A
′	2	$\frac{1}{2}$	$\frac{1}{2}$		
″	3	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	
‴	4		$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$
⁗	5	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{3}{16}$
V	6	$\frac{6}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
VI	7	$\frac{6}{64}$	$\frac{8}{64}$	$\frac{8}{64}$	$\frac{4}{64}$
		&c.			

Sint autem y, z, v, x summæ omnium Probabilitatum quas *B*, *C*, *D*, *A* habent respective ut mulctentur.

Scribantur eodem ordine ac in præcedentibus.

$$B' = B' \qquad\qquad C' = C'$$
$$B'' = B'' \qquad\qquad C'' = C''$$
$$B''' = \tfrac{1}{2}A'' + \tfrac{1}{4}A' \qquad\qquad C''' = \tfrac{1}{2}B'' + \tfrac{1}{4}D'$$
$$B'''' = \tfrac{1}{2}A''' + \tfrac{1}{4}A'' \qquad\qquad C'''' = \tfrac{1}{2}B''' + \tfrac{1}{4}D''$$
$$B^{V} = \tfrac{1}{2}A'''' + \tfrac{1}{4}A''' \qquad\qquad C^{V} = \tfrac{1}{2}B'''' + \tfrac{1}{4}D'''$$
$$B^{VI} = \tfrac{1}{2}A^{V} + \tfrac{1}{4}A'''' \qquad\qquad C^{VI} = \tfrac{1}{2}B^{V} + \tfrac{1}{4}D''''$$

&c. &c.

Ergo $y = \tfrac{3}{4} + \tfrac{1}{2}x + \tfrac{1}{4}x = \tfrac{3}{4} + \tfrac{3}{4}x$. Ergo $z = \tfrac{1}{2} + \tfrac{1}{2}y + \tfrac{1}{4}v$.

Scri-

Scribantur deinde

$D' = D'$
$D'' = D''$
$D''' = \frac{1}{2} C'' + \frac{1}{4} B'$
$D'''' = \frac{1}{2} C''' + \frac{1}{4} B''$
$D^{V} = \frac{1}{2} C'''' + \frac{1}{4} B'''$
$D^{V'} = \frac{1}{2} C^{V} + \frac{1}{4} B''''$
&c.

Ergo $v = \frac{1}{4} + \frac{1}{2} z + \frac{1}{4} y$.

$A' = A'$
$A'' = A''$
$A''' = \frac{1}{2} D' + \frac{1}{4} C'$
$A'''' = \frac{1}{2} D'' + \frac{1}{4} C''$
$A^{V} = \frac{1}{2} D''' + \frac{1}{4} C'''$
$A^{V'} = \frac{1}{2} D^{V} + \frac{1}{4} C''''$
&c

Ergo $x = \frac{1}{2} v + \frac{1}{4} z$.

Resolutis autem quatuor istis æquationibus, invenietur

$y = \frac{243}{249}$ $z = \frac{252}{149}$ $v = \frac{224}{149}$ & $x = \frac{175}{149}$

Ergo si velit Spectator aliquis S mulctas omnes sustinere, æquum erit ut ipsi S

B tradat $\frac{243}{149}p$ C $\frac{252}{149}p$ D $\frac{224}{149}p$ & A $\frac{175}{149}p$.

Sublatis itaque summis probabilitatum quas singuli Collusores habent ut mulctentur, è summis expectationum quas habent iidem si victores abeant, restabunt sortes eorum respective: nempe

B recipit ab R $\frac{4 \times 56}{149} + \frac{45536}{22201} p$

B tradit ipsi S $\frac{243}{149} p$

Ergo ipsi B superest $\frac{224}{149} + \frac{9329}{22201} p$

Sed B deposuerat 1 priusquam ludus inciperetur.

Ergo B lucratur $\frac{75}{149} + \frac{9329}{22201} p$.

 C

C recipit ab *R* $\frac{4 \times 36}{149} + \frac{38724}{22201}p$

C tradit ipsi *S* $\frac{252}{149}p$

Ergo ipsi *C* superest $\frac{144}{149} + \frac{1176}{22201}p$

Sed *C* deposuerat 1.

Ergo *C* lucratur $-\frac{5}{149} + \frac{1176}{22201}p$.

D recipit ab *R* $\frac{4 \times 32}{149} + \frac{37600}{22201}p$

D tradit ipsi *S* $\frac{224}{149}p$

Ergo ipsi *D* superest $\frac{128}{149} + \frac{4224}{22201}p$

Sed *D* deposuerat 1.

Ergo *D* lucratur $-\frac{21}{149} + \frac{4224}{22201}p$.

A recipit ab *R* $\frac{4 \times 25}{149} + \frac{33547}{22201}p$

A tradit ipsi *S* $\frac{175}{149}p$

Ergo ipsi *A* superest $\frac{100}{149} + \frac{7472}{22201}p$

Sed *A* deposuerat $1 + p$, nempe 1 priusquam ludus inchoaretur, & p postquam semel victus fuerat à *B*:

Ergo *A* lucratur $-\frac{49}{149} - \frac{14729}{22201}p$.

Lucrum

Lucrum ipsius $B = +\frac{75}{149} + \frac{9329}{22201}p$

ipsius $C = -\frac{5}{149} + \frac{1176}{22201}p$

ipsius $D = -\frac{21}{149} + \frac{4224}{22201}p$

ipsius $A = -\frac{49}{149} - \frac{14729}{22201}p$

Summa Lucrorum $= \quad 0 \quad 0$

Summa autem lucrorum ipsorum B & $A = \frac{26}{149} - \frac{5400}{22201}p$; sed posueramus B vicisse ipsum A semel, priusquam Collusores pacta inirent cum R & S. Priusquam vero ludus inchoaretur, A poterat æqua sorte expectare ut vinceret ipsum B, adeoque summa lucrorum $\frac{26}{149} - \frac{5400}{22201}$ in duas partes æquales est dividenda, ita ut utriusque lucrum censendum sit $\frac{13}{149} - \frac{2700}{22201}p$.

Ponatur $\frac{13}{149} - \frac{2700}{22201}p = 0$, & erit $p = \frac{1937}{2700}$.

Ergo si sit mulcta p ad summam quam singuli deponunt ut 1937 ad 2700, A & B nihil lucrantur, nihil perdunt. Verum hoc in Casu C lucratur $\frac{1}{225}$, quam D perdit.

Coroll. 1. Spectator R, priusquam ludus inchoetur, id suscipere in se poterit, ut summam 4 de qua Collusores contendunt, & mulctas omnes pendat, si sibi initio in manus darentur $4 + 7p$.

Coroll. 2. Si dexteritates Collusorum sint in ratione data, sortes Collusorum eadem ratiocinatione determinabuntur.

Coroll. 3. Si Series aliqua ita ſit conſtituta, ut continuò decreſcat, & terminus quivis ad præcedentes quoſlibet habeat rationes datas, ſive eaſdem ſive diverſas, ſeries iſta accurate ſummabitur. Inſuper ſi termini omnes hujus Seriei multiplicentur per terminos progreſſionis Arithmeticæ, ſinguli per ſingulos, Series nova reſultans accurate ſummabitur.

Coroll 4. Si ſint Series plures collaterales, ita relatæ ut terminus quilibet cujuſque Seriei ad præcedentes quoſlibet aliarum Serierum habeat rationes datas, ſive eaſdem ſive diverſas, ita ut Series iſtæ collaterales ſe decuſſent data qualibet lege conſtanti, Series iſtæ accurate ſummabuntur. Inſuper ſi termini omnes harum Serierum multiplicentur ordinatim per terminos Progreſſionis Arithmeticæ, ſinguli per ſingulos, Series novæ ex hac multiplicatione reſultantes etiamnum accurate ſummabuntur.

Clavis ad Problema generale.

Si ſint Colluſores quotcunque *v. g.* Sex, B, C, D, E, F, A & Probabilitates quas habent ut victores evadant, ſive ut mulctentur, dato Ludorum numero, denotentur reſpective B, C, D, E, F & A; & Probabilitates dato Ludorum numero his proximo & minori competentes, per $B_{\prime\prime}$, $C_{\prime\prime}$, $D_{\prime\prime}$, $E_{\prime\prime}$, $F_{\prime\prime}$, $A_{\prime\prime}$; & Probabilitates dato Ludorum numero his itidem noviſſimis proximo & minori competentes, per $B_{\prime\prime\prime}$, $C_{\prime\prime\prime}$, $D_{\prime\prime\prime}$, $E_{\prime\prime\prime}$, $F_{\prime\prime\prime}$, $A_{\prime\prime\prime}$, & ſic deinceps; erit ſemper,

$$
\begin{aligned}
B_{\prime} &= \tfrac{1}{2}A_{\prime\prime} + \tfrac{1}{4}A_{\prime\prime\prime} + \tfrac{1}{8}A_{\prime\prime\prime\prime} + \tfrac{1}{16}A_{V} \\
C_{\prime} &= \tfrac{1}{2}B_{\prime\prime} + \tfrac{1}{4}F_{\prime\prime\prime} + \tfrac{1}{8}E_{\prime\prime\prime\prime} + \tfrac{1}{16}D_{V} \\
D_{\prime} &= \tfrac{1}{2}C_{\prime\prime} + \tfrac{1}{4}B_{\prime\prime\prime} + \tfrac{1}{8}F_{\prime\prime\prime\prime} + \tfrac{1}{16}E_{V} \\
E_{\prime} &= \tfrac{1}{2}D_{\prime\prime} + \tfrac{1}{4}C_{\prime\prime\prime} + \tfrac{1}{8}B_{\prime\prime\prime\prime} + \tfrac{1}{16}F_{V} \\
F_{\prime} &= \tfrac{1}{2}E_{\prime\prime} + \tfrac{1}{4}D_{\prime\prime\prime} + \tfrac{1}{8}C_{\prime\prime\prime\prime} + \tfrac{1}{16}B_{V} \\
A_{\prime} &= \tfrac{1}{2}F_{\prime\prime} + \tfrac{1}{4}E_{\prime\prime\prime} + \tfrac{1}{8}D_{\prime\prime\prime\prime} + \tfrac{1}{16}C_{V}
\end{aligned}
$$

Et fiat ſemper retrogreſſus ordinatim ad tot literas minus duobus quot ſunt Colluſores, omittaturque ſemper litera A, prima æquatione excepta, ubi litera A terminos omnes præter primam occupat.

IV.

Intrat. *Exit.*

Depositum		Sors	Depositum		Sors	Deposit.	
$n+1$	0	Z	$n+1+p$	1	H	$n+1+$	$A=Z$
$n+1+p$	1	Y	$n+1+2p$	2	K	$n+1+p$	$C=Y$ N.° 1
$n+1+2p$	2	X	$n+1+3p$	3	L	$n+1+2p$	$D=\frac{1}{2}X+\frac{1}{2}\times\overline{Y+yp}$
$n+1+3p$	3	V	$n+1+4p$	4	M	$n+1+3p$	$E=\frac{1}{4}V+\frac{1}{4}\times\overline{X+xp}+\frac{1}{2}\times\overline{Y+2yp}$
$n+1+4p$	4	T				$n+1+4p$	$F=\frac{1}{8}T+\frac{1}{8}\times\overline{V+up}+\frac{1}{4}\times\overline{X+2xp}+\frac{1}{2}\times\overline{Y+3yp}$

N.° 2

$$=\tfrac{1}{2}\times\overline{H-p}+\tfrac{1}{4}\times\overline{H-p+hp}+\tfrac{1}{8}\times\overline{H-p+2hp}+\tfrac{1}{16}\times\overline{H-p+3hp}+\cdots\tfrac{1}{2^n}\times\overline{H-p+nhp-hp}+\tfrac{1}{2^n}\times\overline{np+n+1}$$

$$=\tfrac{1}{2}\times\overline{K-p}+\tfrac{1}{4}\times\overline{H-p+2hp}+\tfrac{1}{8}\times\overline{H-p+3hp}+\tfrac{1}{16}\times\overline{H-p+4hp}+\cdots\tfrac{1}{2^n}\times\overline{H-p+nhp}+\tfrac{1}{2^n}\times\overline{np+p+n+1}$$

$$=\tfrac{1}{2}\times\overline{L-p}+\tfrac{1}{4}\times\overline{H-p+3hp}+\tfrac{1}{8}\times\overline{H-p+4hp}+\tfrac{1}{16}\times\overline{H-p+5hp}+\cdots\tfrac{1}{2^n}\times\overline{H-p+nhp+hp}+\tfrac{1}{2^n}\times\overline{np+2p+n+1}$$

$$=\tfrac{1}{2}\times\overline{M-p}+\tfrac{1}{4}\times\overline{H-p+4hp}+\tfrac{1}{8}\times\overline{H-p+5hp}+\tfrac{1}{16}\times\overline{H-p+6hp}+\cdots\tfrac{1}{2^n}\times\overline{H-p+nhp+hp}+\tfrac{1}{2^n}\times\overline{np+3p+n+1}$$

N.° 3

$$=\tfrac{1}{2^{n-1}}\times\overline{C+ncp-cp}+\tfrac{1}{2^{n-2}}\times\overline{D+ndp-2dp}+\tfrac{1}{2^{n-3}}\times\overline{E+nep-3ep}+\tfrac{1}{2^{n-4}}\times\overline{F+nfp-4fp}+\cdots$$

$$=\tfrac{1}{2^{n-2}}\times\overline{D+ndp-dp}+\tfrac{1}{2^{n-3}}\times\overline{E+nep-2ep}+\tfrac{1}{2^{n-4}}\times\overline{F+nfp-3fp}+\cdots$$

$$=\tfrac{1}{2^{n-3}}\times\overline{E+nep-ep}+\tfrac{1}{2^{n-4}}\times\overline{F+nfp-2fp}+\cdots$$

$$=\tfrac{1}{2^{n-4}}\times\overline{F+nfp-fp}+\cdots$$

N.° 4 N.° 6 N.° 8

$$-Z=\tfrac{1}{2}K-\tfrac{1}{2}H+\tfrac{1}{4}hp+\tfrac{1}{8}hp+\tfrac{1}{16}hp+\cdots\tfrac{1}{2^n}\times hp=\tfrac{1}{2}K-\tfrac{1}{2}H+zp-\tfrac{1}{2}hp=-\tfrac{1}{2^{n-0}}\times C-\tfrac{ncp}{2^n}+zp$$

$$-Y=\tfrac{1}{2}L-\tfrac{1}{2}K+\tfrac{1}{4}hp+\tfrac{1}{8}hp+\tfrac{1}{16}hp+\cdots\tfrac{1}{2^n}\times hp=\tfrac{1}{2}L-\tfrac{1}{2}K+zp-\tfrac{1}{2}hp=-\tfrac{1}{2^{n-1}}\times D-\tfrac{ndp}{2^{n-1}}-\tfrac{cp}{2^n}+zp$$

$$-X=\tfrac{1}{2}M-\tfrac{1}{2}L+\tfrac{1}{4}hp+\tfrac{1}{8}hp+\tfrac{1}{16}hp+\cdots\tfrac{1}{2^n}\times hp=\tfrac{1}{2}M-\tfrac{1}{2}L+zp-\tfrac{1}{2}hp=-\tfrac{1}{2^{n-2}}\times E-\tfrac{nep}{2^{n-2}}-\tfrac{dp}{2^{n-1}}-\tfrac{cp}{2^n}+zp$$

N.° 5 N.° 7

$$-H=-\tfrac{1}{2^{n-1}}\times\overline{C+ncp-cp}+\tfrac{1}{2^{n-2}}\times dp+\tfrac{1}{2^{n-3}}\times ep+\tfrac{1}{2^{n-4}}\times fp+\cdots=-\tfrac{1}{2^{n-1}}\times C-\tfrac{ncp}{2^{n-1}}+hp$$

$$-K=-\tfrac{1}{2^{n-2}}\times\overline{D+ndp-dp}+\tfrac{1}{2^{n-3}}\times ep+\tfrac{1}{2^{n-4}}\times fp+\cdots=-\tfrac{1}{2^{n-2}}\times D-\tfrac{ndp}{2^{n-2}}-\tfrac{cp}{2^{n-1}}+hp$$

$$-L=-\tfrac{1}{2^{n-3}}\times\overline{E+nep-ep}+\tfrac{1}{2^{n-4}}\times fp+\cdots=-\tfrac{1}{2^{n-3}}\times E-\tfrac{nep}{2^{n-3}}-\tfrac{dp}{2^{n-2}}-\tfrac{cp}{2^{n-1}}+hp$$

N.° 9.

$=A$

$=C$

$=2D-C-cp$

$=4E-2D-C-2dp-2cp$

N.° 10 N.° 8

$$C-A=Y-Z=-\tfrac{1}{2^n}\times C-\tfrac{ncp}{2^n}+zp$$

$$2D-2C-cp=X-Y=-\tfrac{1}{2^{n-1}}\times D-\tfrac{ndp}{2^{n-1}}-\tfrac{cp}{2^n}+zp$$

$$4E-4D-2dp-cp=V-X=-\tfrac{1}{2^{n-2}}\times E-\tfrac{nep}{2^{n-2}}-\tfrac{dp}{2^{n-1}}-\tfrac{cp}{2^n}+zp$$

N.° 11 N.° 12

$$=\frac{A\times 2^n+ap\times 2^n-ncp}{1+2^n}=\frac{\overline{A+ap}\times 2^n-ncp}{1+2^n}$$

$$D=\frac{C\times 2^n+cp\times\overline{2^{n-1}-\frac{1}{2}}+ap\times 2^{n-1}-ndp}{1+2^n}=\frac{\overline{C+cp}\times 2^n-ndp}{1+2^n}$$

$$E=\frac{D\times 2^n+dp\times\overline{2^{n-1}-\frac{1}{2}}+cp\times\overline{2^{n-2}-\frac{1}{4}}+ap\times 2^{n-2}-nep}{1+2^n}=\frac{\overline{D+dp}\times 2^n-nep}{1+2^n}$$

I. Senex sculp.t